

That Darn Bouncing Ball Problem

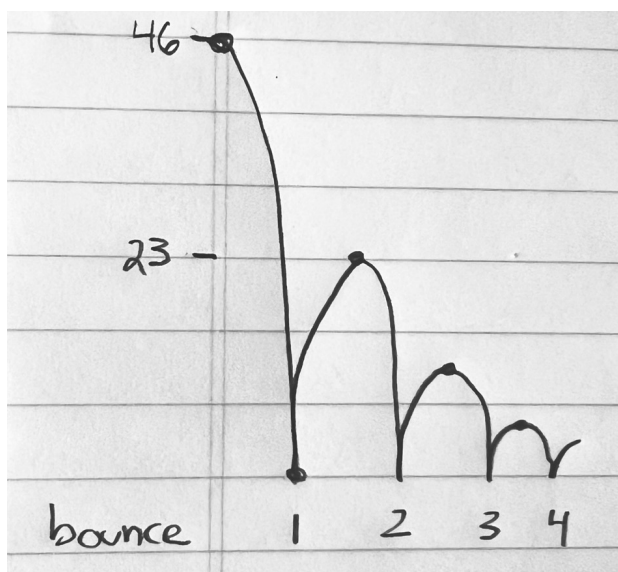
Skills needed:

Knowledge of the partial sums of a geometric series formula

* Names have been changed to protect the identity of the persons in this problem

Martha dropped a ball from a height of 46 yards. After each bounce, the peak height of the ball is half the peak height of the previous height. How far does the ball travel from the time Martha drops the ball until it reaches its peak height after the 4th bounce?

Here is a visual representation of the bouncing ball:



The solution is applying the partial sums for a geometric series formula twice.

The formula is $S_n = \frac{a_1(1-r^n)}{1-r}$ where a_1 is the first term, n is the number of terms to sum, and r is the common ratio between the terms. The sequence of heights of the ball are as follows:

Downward travel distances: $a = \{46, 23, 11.5, \dots\}$ where a_1 is 46 and $r = 0.5$

Upward travel distances: $a = \{23, 11.5, 5.75, \dots\}$ where a_1 is 23 and $r = 0.5$

Solution. Apply the partial sums formula to each set of distances, both the 'upward' and 'downward' travel paths; add the two partial sums together. Using [Desmos](#),

$$\frac{46(1-0.5^4)}{1-0.5} + \frac{23(1-0.5^4)}{1-0.5} = 129.375$$