ALGEBRA

PROPERTIES

ARITHMETIC PROPERTIES

ASSOCIATIVE	$a(bc) = (ab)c$
COMMUTATIVE	$a + b = b + a$ and $ab = ba$
DISTRIBUTIVE	$a(b + c) = ab + ac$

ARITHMETIC OPERATIONS EXAMPLES

$$ab + ac = a(b + c)$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c}$$

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab + ac}{a} = b + c, a \neq 0$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

QUADRATIC EQUATION

For the equation

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RADICAL PROPERTIES

$a, b \geq 0$ for even n

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

LOGARITHM PROPERTIES

if $y = \log_b x$ then $b^y = x$

$\log_b b = 1$ and $\log_b 1 = 0$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_b(x^r) = r \log_b x$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

EXPONENT PROPERTIES

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$a^0 = 1, a \neq 0$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{1}{a^{-n}} = a^n$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$$

PROPERTIES OF INEQUALITIES

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $a/c < b/c$

If $a < b$ and $c < 0$ then $ac > bc$ and $a/c > b/c$

PROPERTIES OF COMPLEX NUMBERS

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$\sqrt{-a} = i\sqrt{a}, \quad a \geq 0$

$$(a + bi) + (c + di) = a + c + (b + d)i$$

$$(a + bi) - (c + di) = a - c + (b - d)i$$

$$(a + bi)(c + di) = ac - bd + (ad + bc)i$$

$$(a + bi)(a - bi) = a^2 + b^2$$

$$|a + bi| = \sqrt{a^2 + b^2}$$

$$\overline{(a + bi)} = a - bi$$

$$\overline{(a + bi)}(a + bi) = |a + bi|^2$$

$$\frac{1}{(a + bi)} = \frac{(a - bi)}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2}$$

COMMON FACTORING EXAMPLES

$$x^2 - a^2 = (x + a)(x - a)$$

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x + a)^3$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$

ABSOLUTE VALUE

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

$$|a| = |-a|$$

$$|a| \geq 0$$

$$|ab| = |a||b|$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$|a + b| \leq |a| + |b|$$

COMPLETING THE SQUARE

$$ax^2 + bx + c = a(\ldots)^2 + \text{constant}$$

1. Divide by the coefficient a.
2. Move the constant to the other side.
3. Take half of the coefficient b/a, square it and add it to both sides.
4. Factor the left side of the equation.
5. Use the square root property.
6. Solve for x.

TRIGONOMETRY

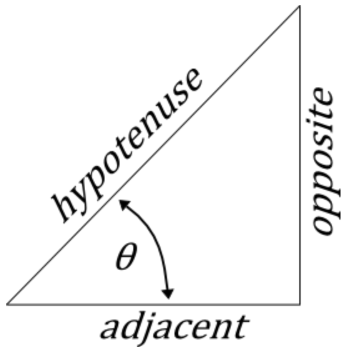
DEFINITION

RIGHT TRIANGLE DEFINITION

$\sin\theta = \dfrac{opposite}{hypotenuse}$ $\csc\theta = \dfrac{hypotenuse}{opposite}$

$\cos\theta = \dfrac{adjacent}{hypotenuse}$ $\sec\theta = \dfrac{hypotenuse}{adjacent}$

$\tan\theta = \dfrac{opposite}{adjacent}$ $\cot\theta = \dfrac{adjacent}{opposite}$



TRIG FUNCTIONS RANGE

$-1 \le \sin\theta \le 1$

$-1 \le \cos\theta \le 1$

$-\infty \le \tan\theta \le \infty$

$\csc\theta \ge 1$ and $\csc\theta \le -1$

$\sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty \le \cot\theta \le \infty$

UNIT CIRCLE DEFINITION

$\sin\theta = y$

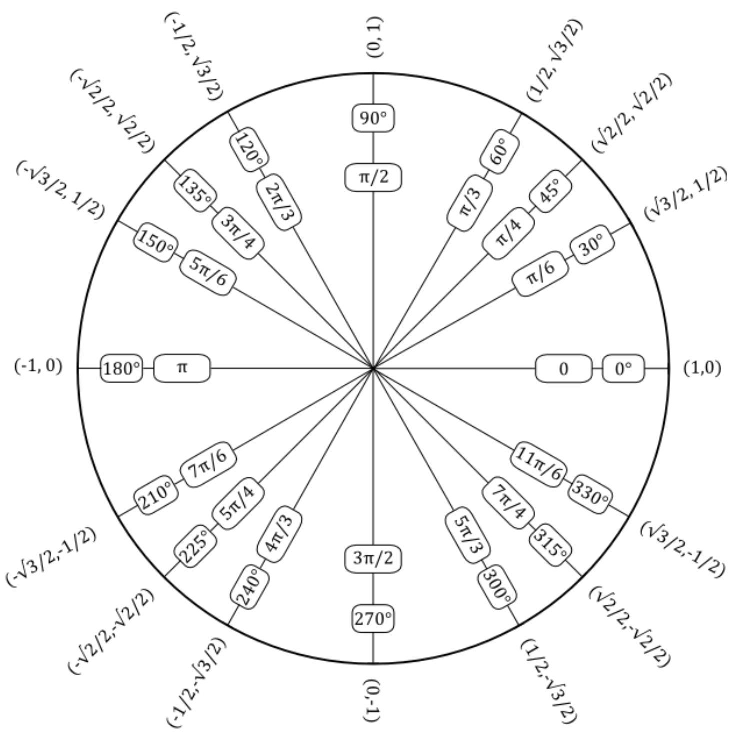
$\cos\theta = x$

$\tan\theta = \dfrac{y}{x}$

$\csc\theta = \dfrac{1}{y}$

$\sec\theta = \dfrac{1}{x}$

$\cot\theta = \dfrac{x}{y}$



TRIG FUNCTIONS DOMAIN

$\sin\theta,\ \theta$ can be any angle

$\cos\theta,\ \theta$ can be any angle

$\tan\theta,\ \theta \ne \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm1, \pm2, \ldots$

$\csc\theta,\ \theta \ne n\pi, \quad n = 0, \pm1, \pm2, \ldots$

$\sec\theta,\ \theta \ne \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm1, \pm2, \ldots$

$\cot\theta,\ \theta \ne n\pi, \quad n = 0, \pm1, \pm2, \ldots$

TRIG FUNCTIONS PERIOD

$\sin(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\cos(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\tan(\omega\theta) \rightarrow T = \dfrac{\pi}{\omega}$

$\csc(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\sec(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\cot(\omega\theta) \rightarrow T = \dfrac{\pi}{\omega}$

INVERSE TRIG FUNCTION NOTATION

$\sin^{-1} x \equiv \arcsin x \equiv \text{Asin}\, x$

$\cos^{-1} x \equiv \arccos x \equiv \text{Acos}\, x$

$\tan^{-1} x \equiv \arctan x \equiv \text{Atan}\, x$

INVERSE TRIG DOMAIN

$\sin^{-1} x : -1 \le x \le 1$

$\cos^{-1} x : -1 \le x \le 1$

$\tan^{-1} x : -\infty \le x \le \infty$

INVERSE TRIG FUNCTION RANGE

$-\dfrac{\pi}{2} \le \sin^{-1} x \le \dfrac{\pi}{2}$

$0 \le \cos^{-1} x \le \pi$

$-\dfrac{\pi}{2} \le \tan^{-1} x \le \dfrac{\pi}{2}$